Useful Life for a Panel with Intermediate Supports Undergoing Creep

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Nomenclature

A, B, k = const for creep strain equation
D = stiffness coeff
E' = $E(1 - \mu)/(1 - \mu^2)$
h = thickness of panel
K = spring const for the support
P = applied load
w = deflection of the median surface
w'' = inelastic deflection of the support
w_m = w at $x = L/2$
w_0 = initial deflection of the median surface
ϵ'' = inelastic strain
η = distance from neutral axis

Introduction

In order to estimate the useful life of a panel with intermediate supports, such as the exterior skin of a hypersonic airplane wing, it is necessary to know its time dependent deflection behavior under compressive loads at high temperature. This requirement also could apply to a stiffened panel with stiffeners resting on the considerably more rigid interior skin. There are a number of studies concerning the creep problem of plates. They are based on either initially imperfect plates or initially perfect ones. In this study, because of the more realistic nature of the problem, the panel is assumed to have an initial imperfection. A time increment method is used similar to the one used in Ref. 1. Finally, a relationship between the useful life of the panel and the time of deceleration is pointed out and suggested as a possible criterion for the creep allowable time.

Description of the method used

The solution consisted of the following steps: 1) Considering the plate as a skin panel resting on the main structure, which is not subjected to elevated temperatures, it was idealized as a simply supported plate with some initial curvature resting on supports, where both the plate and the supports are subjected to creep (see Fig. 1); 2) for the finite difference equations, a grid system with node points over the support system was used; 3) assuming that during a time increment stresses remain constant, equilibrium equations in the direction normal to the median surface were written for every node point of the grid, and the resulting set of equations was solved by means of computer; 4) inelastic deformations occurring in the plate and in the supports were transformed into lateral loads acting on the node points, and step 3 was repeated after each time increment.

The following assumptions were made in order to facilitate the computation: at the time zero, the imperfection of the plate is given as $w_0 = a \sin \Pi X/L$, at any time w_{yy}, and w_{xy}, are negligible compared to $w_{,xx}$. The compressive load acts only in the direction normal to the support direction.

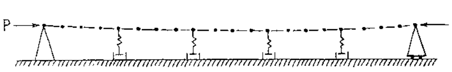

Fig. 1 Idealized panel as used for finite difference equations.

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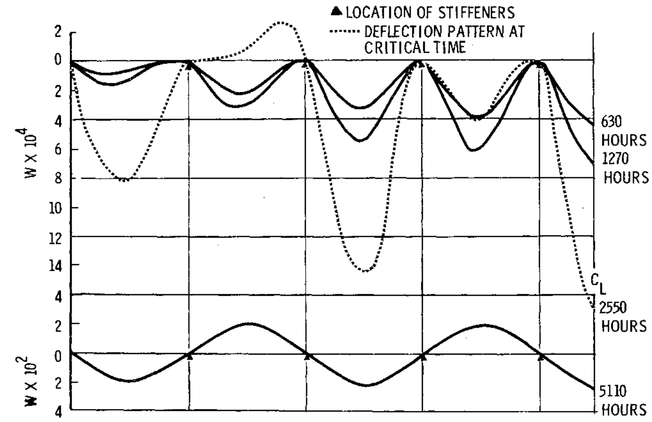

Fig. 2 Time dependent deflection pattern for a stiffened panel under uniaxial compression load (even number of stiffeners).

Formulation of the problem

The equilibrium equation is given as

$$Dw_{,xxxx} + Pw_{,xx} + Kw = \left(E' \int_{-h/2}^{h/2} \epsilon'' \eta d\eta\right)_{,xx} - Pw_{0,xx} + Kw'' \quad (1)$$

where the terms of the forcing function are obtained by transforming the inelastic strains, the initial deflection, and the inelastic settlement of the supports into lateral loads. Writing in the form of difference equations, we obtain

$$D(\Delta x)^{-4}(w_{n+2} - 4w_{n+1} + 6w_n - 4w_{n-1} + w_{n-2}) + P(\Delta x)^{-2}(w_{n+1} - 2w_n + w_{n-1}) + K_n w_n = E' (\Delta x)^{-2} [(\Sigma \epsilon'' \eta \Delta \eta)_{n+1} - 2(\Sigma \epsilon'' \eta \Delta \eta)_n + (\Sigma \epsilon'' \eta \Delta \eta)_{n-1}] - P(\Delta x)^{-2}(w_{0n+1} - 2w_{0n} + w_{0n-1}) + K_n w_n'' \quad (2)$$

where K_n is nonzero only at the node points corresponding to support locations.

After the application of the load, at time O^+, all terms of the forcing function are zero except the one corresponding to the initial deflection. By solving the simultaneous equations, the w_n values are found. The stress levels and the loads acting on the supports are computed by means of Eq. (3).

$$\sigma(X_1, \eta, O^+) = E\eta\, w_{,xx} \qquad R_n = K_n w_n \quad (3)$$

Assuming the stress levels remain constant during the following time increment, the creep strains in the plate and the creep deflections at the supports are computed using the stress-strain-time relations given [Eq. (8)]. This information then is fed back to Eq. (2) and the deflection value corresponding to the end of the first time increment is evaluated.

At the beginning of the second time increment, stresses are recomputed using Eq. (4).

$$\sigma(x, \eta, \Delta t) = E [h^{-1} \Sigma \epsilon'' \Delta \eta + \eta(\Delta x^{-2})(w_{n+1} - 2 w_n + w_{n-1}) - \Delta \epsilon''] \quad (4)$$

With the new stress levels, the creep strains and deformations are reevaluated and the procedure is repeated for the following time increments.

Stress-strain-time relations

A suitable formula for the creep strain is given in Ref. 2 as

$$\epsilon'' = A (\sinh B\, \sigma_e) t^k \quad (5)$$

For the material with some existing creep strain, the real time can be converted into a fictitious time based on the third law of creep.[3] This law indicates that the amount of incremental creep which will take place in the material depends only on the

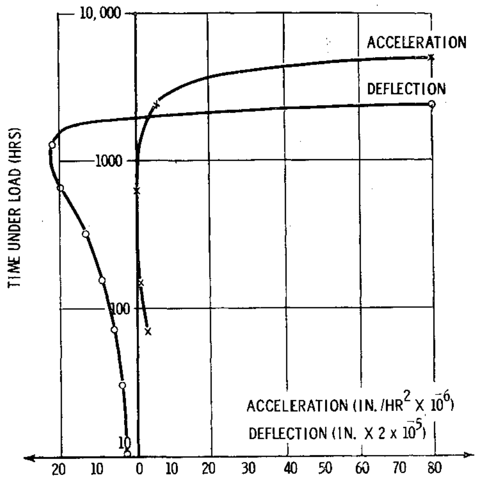

Fig. 3 Vertical deflection and acceleration vs time for a point located at 14/32 *L* distance from the edge.

stress level, the amount of existing creep strain, the temperaperature, and the duration of the time increment that the stress level remains constant.

Due to the incremental nature of the procedure, the strain at the end of ith increment is represented as

$$\epsilon'' = C_i t_i^{k_i} \text{ where } C_i = A_i \sinh B\sigma_i \text{ and } t_i = t_{i-1}^* + \Delta t_i \quad (6)$$

Using a recurrence relationship, each t_i^* also is represented in terms of the previous ones:

$$t_i^* = C_1^{1/k_{i+1}} C_{i+1}^{-1/k_{i+1}} (t_{i-1}^* + \Delta t_i^*)^{k_i/k_{i+1}} \quad (7)$$

which, in turn, leads to the elimination of the t^* from Eq. (6). In the example problem, temperature remains constant during the life of the structure, and time increments are taken such that the logarithmic time increment goes to a constant value as i increases. Then the resulting creep strain is

$$\epsilon_i'' = \left(\Delta t \sum_i C_i^{1/k} \rho^{i-1}\right)^k \quad \text{where} \quad \rho = \Delta t_{i+1}/\Delta t_i \quad (8)$$

Discussion of Results

The behavior of the integrally stiffened panel as idealized in this study suggested two interesting points: 1) the possibility of determining the time-dependent deflection pattern of either a stiffened panel or a panel with intermediate supports by a simple numerical procedure, and 2) the possibility of establishing a safe allowable creep life for compression instability. As indicated in Ref. 4, the existing creep buckling data suggest a relation between the $w_{,tt}$ and the allowable compression life of a column. In Ref. 5, Gerard suggested the $w_{m,t}/w_m = 0$, following an infinitesimal lateral disturbance applied to a perfect column, as the stability criterion for creep buckling. In the case of initial imperfectness, there is no need for the infinitesimal lateral disturbance for the purpose of inspection and, if the time of $w_{m,tt} = 0$ is taken as the limit of the useful life, the column is considered safe to carry the load as long as it keeps deflecting at a decreasing rate.

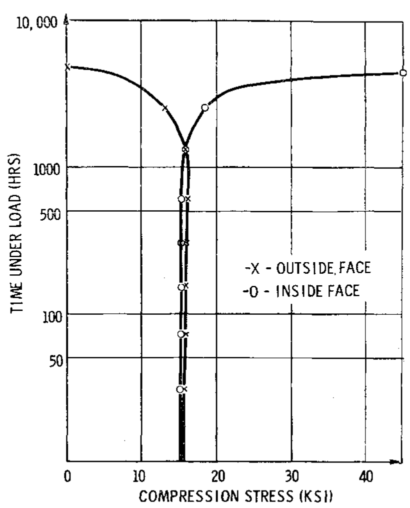

Fig. 4 Stress on panel faces vs time at a point located at 14/32 *L* distance from the edge.

Figures 2 and 3 demonstrate the time-dependent deflection pattern, and the relation between the time of zero deceleration and the time required to reach the stage of unbound deflection. In Fig. 4, the relation between the stress levels on the concave and the convex surfaces and the time under compressive load has been shown. This can be explained by observing that the material on the concave side of the panel is under the influence of two factors. The first one is the amount of creep strain, and, due to the strain hardening, creep rate reduces as the creep strain increases. The second factor is the acting stress that increases as the curvature increases, thus increasing the creep rate. During the time period immediately following the application of the load, the influence of the strain hardening is more pronounced and it causes the panel to deflect at a decreasing rate. At time $w_{m,tt} = 0$, the influence of these two factors balance each other, and from then on, the panel deflects at an ever increasing rate and finally collapses. Therefore, it can be concluded that the time of zero deceleration could have been taken as the safe allowable time for the compression life of the panel.

Appendix A

The numerical results shown in Figs. 2 and 3 are obtained with the following values of constants: L = 24.00 in.; a = 0.0123 in.; h = 0.1230; support height = 1.0 in.; support width = 0.25 in.; $E = 9.4 \times 10^6$ psi; $\mu = 0.3$; $A = 6.8 \times 10^{-5}$; $B = 9300^{-1}$ psi; $k = 0.5$; applied compressive load = 1900 lb/in. The material is 2024-T3 aluminum alloy at 400°F.

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Frequency and Damping from Time Histories: Maximum-Slope Method

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Introduction

DETERMINATION of the undamped natural frequency and damping ratio of highly damped second-order systems from time history data is notoriously difficult because of the lack of measurable response overshoot. The maximum-slope method is an easily-used technique developed specifically for systems having damping ratios of 0.5 to 1.4. It does not

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